6[K, X]. V. V. Solodovnikov, *Introduction to the Statistical Dynamics of Automatic Control Systems*, Translation edited by John B. Thomas and Lotfi A. Zadeh, Dover Publications, New York, 1960, xx + 307 p., 20 cm. Price $2.25 (Paperbound).

This book, first published in Russian in 1952, gives an excellently written, self-contained account of the principles of the analysis of linear systems, the statistics of random signals, and the theory of linear prediction and filtering. The translation is well done. In addition to treating exact methods, the author discusses methods of obtaining approximate solutions to various problems.

The first three chapters are devoted to a discussion of the theory of the transients in a linear system produced by deterministic signals, to the elements of probability theory, and to the basic concepts of the theory of stationary random processes.

Chapter IV discusses the criterion of least mean-square error. Linear and square-law detectors are used to show how some nonlinear systems may be treated.

In Chapter V the problem of using numerical methods to approximate spectral distribution curves is treated.

Chapters VI, VII, and VIII contain the derivation and application of formulas from which one may obtain the transfer function yielding a minimum mean-square error from the knowledge of the spectral densities of the signal and noise. The last of these chapters treats the case where the signal is composed of two parts, one deterministic and one random.

The book contains four appendices. Appendix I consists of five-place tables of the functions $\frac{\sin x}{x}$ and $\frac{\cos x}{x}$ for $x = 0(.01)10.0(.1)20(1)100$. Appendix II contains tables of the first five Laguerre functions to five significant figures for values of the argument in the range 0(.01)1.0(.1)20(1)30. Appendices IIIa and IIIb give five-place tables for the calculation of the so-called phase characteristic function from straight-line approximations of the logarithm of the spectral-density function. Appendix IV gives a table of integrals

$$I_n = \frac{1}{2\pi j} \int_{-\infty}^{\infty} \frac{G_n(jw)}{H_n(jw)H_n(-jw)}\, dw, \; n = 1(1)7,$$

where

$$G_n(jw) = b_0(jw)^n + b_1(jw)^{n-1} + \cdots + b_n,$$
$$H_n(jw) = A_0(jw)^n + A_1(jw)^{n-1} + \cdots + A_n,$$

and all roots of $H_n(jw)$ are in the upper half-plane.

A. H. T.

7[L]. David J. BenDaniel & William E. Carr, *Tables of Solutions of Legendre's Equation for Indices of Nonintegral Order*, University of California Lawrence Radiation Laboratory, Livermore, UCRL-5859, September, 1960, 68 p., 28 cm. Available from the Office of Technical Services, Washington 25, D. C. Price $1.75.

We employ the usual notation for hypergeometric and Legendre functions [1]. Let

$$f_1(x) = {}_2F_1(-\nu/2, \nu/2 + \tfrac{1}{2}; \tfrac{1}{2}; x^2); \quad f_2(x) = x\,{}_2F_1(\tfrac{1}{2} - \nu/2, 1 + \nu/2; 3/2; x^2).$$

Then $f_1(x)$ and $f_2(x)$ satisfy the differential equation

$$(x^2 - 1)\, d^2f/dx^2 + 2x\, df/dx - \nu(\nu + 1)f = 0,$$

and we have

$$P_\nu(x) = a_1 f_1(x) + a_2 f_2(x), \qquad Q_\nu(x) = b_1 f_1(x) + b_2 f_2(x),$$

where

$$a_1 = \pi^{1/2}[\Gamma(\tfrac{1}{2} - \nu/2)\Gamma(1 + \nu/2)]^{-1}, \quad a_2 = -2\pi^{1/2}[\Gamma(\tfrac{1}{2} + \nu/2)\Gamma(-\nu/2)]^{-1}$$

$$b_1 = -\frac{\tfrac{1}{2}\pi^{1/2}\Gamma(\tfrac{1}{2} + \nu/2)\sin \nu\pi/2}{\Gamma(1 + \nu/2)}, \quad b_2 = \frac{\pi^{1/2}\Gamma(1 + \nu/2)\cos \nu\pi/2}{\Gamma(\tfrac{1}{2} + \nu/2)}.$$

Table 1 gives $f_1(x)$ to 5S, corresponding to $x = 0(0.01)0.99$, $\nu = 0(0.0625)1(0.125)10(0.25)36$.

For a given ν, let $\alpha_i^{(\nu)}$ be the i-th zero of $f_1(x)$. Then Table 2 gives 4S values of $\alpha_i^{(\nu)}$ and of the integrals

$$\int_0^{\alpha_i^{(\nu)}} f_1(x)\, dx, \quad \int_0^{\alpha_i^{(\nu)}} \{f_1(x)\}^2\, dx, \quad \text{for} \quad \nu = 0.4375(0.0625)36.$$

Tables 3 and 4 present corresponding data for $f_2(x)$.

The tables are essentially new, and were obtained on an automatic computer. An introduction gives a few definitions (the coefficients a_1 and a_2 in the formula for $P_\nu(x)$ contain typographical errors). There is no discussion of formulas used to perform and check the calculations. No attempt is made to give closed-form results, which would be useful to the applied worker. For example, we can show that

$$f_1(x) = (\cos\theta/2)^{-1/2} \cos\left\{N_1\theta + \frac{1}{16N_1}(\tan\theta/2 + \theta/2)\right\}\{1 + O(1/\nu^2)\},$$

$$\int_0^x f_1(t)\, dt = (2N_1)^{-1}(\cos\theta/2)^{1/2} \sin\left\{N_1\theta + \frac{1}{16N_1}(-3\tan\theta/2 + \theta/2)\right\} \cdot\{1 + O(1/\nu^2)\},$$

$$f_2(x) = (2N_2)^{-1}(\cos\theta/2)^{-1/2} \sin\left\{N_2\theta + \frac{1}{16N_2}(\tan\theta/2 + 9\theta/2)\right\} \cdot\{1 + O(1/\nu^2)\},$$

$$\int_0^x f_2(t)\, dt = -(2N_2)^{-2}(\cos\theta/2)^{1/2} \cos\left\{N_2\theta + \frac{3}{16N_2}(-\tan\theta/2 + 3\theta/2)\right\} \cdot\{1 + O(1/\nu^2)\}$$

$$N_1^2 = \nu(\nu + 1)/4, \quad N_2^2 = (\nu - 1)(\nu + 2)/4, \quad x = \sin\theta/2, \quad 0 \leqq \theta < \pi.$$

Indeed, using these results, we have spot checked the entries. Tables 1 and 3 and values of the zeros of f_1 and f_2 in Tables 2 and 4 appear to be correct.

If $\nu = 20$, and $\alpha = 1, 2, 3$, the values of $\int_0^{x_\alpha} f_1(t)\, dt$ are erroneous, as is also the value of $\int_0^{x_1} f_1^2(t)\, dt$. It is curious that the error in each case is about 0.0021.

Similar check calculations for $f_2(x)$ reveal a persistent error of about 0.0024. Thus the tables of the numerical values of the integrals should be used, if at all, with caution. We have corresponded with one of the authors (D.J.B.). He has checked those entries in Tables 1 and 3 against known values of Legendre polynomials and finds that they are correct. The reason for the bias in the values of the integrals is not known, but he suspects that it arises from the binary-to-decimal conversion. We conclude with the "trite" observation that automatic computers cannot be trusted implicitly, and that the need for analysis and checking remains.

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1. A. Erdélyi, et al., *Higher Transcendental Functions*, Vol. 1, McGraw-Hill, New York, 1953.

8[L]. Ludo K. Frevel & J. W. Turley, *Tables of Iterated Sine-Integral*, The Dow Chemical Company, Midland, Michigan, 1961. Deposited in UMT File.

Three tables of decimal values of the iterated sine-integral, $\overset{n}{Si}(x)$, are herein presented, as computed on a Burroughs 220 system supplemented by Cardatron equipment, which permitted on-line printing of the results in the desired tabular format.

Table 1 presents the values of $\overset{n}{Si}(x)$ to 9D for $n = 1(1)10$, $x = 0(0.2)10$. Table 2 gives values of this function to 7D for $n = 0(0.05)10$, π, 2π, 3π, and Table 3 gives for $n = 1(1)10$ the values to 9D of the first thirty extrema, which correspond to $x = m\pi$, where $m = 1(1)30$.

In an accompanying text of three pages the authors describe in detail the method of calculation and the underlying mathematical formulas. It is there stated that the entries in Table 2 were computed to 9D prior to rounding. The entries in Table 3 are claimed to be accurate to within a unit in the final decimal place, and the authors imply in their explanatory text that comparable accuracy was attained in the computation of the entries in Table 1.

The tabular data corresponding to the values of n different from unity constitute an original contribution to the literature of mathematical tables.

J. W. W.

9[L, X]. Hans Sagan, *Boundary and Eigenvalue Problems in Mathematical Physics*, John Wiley & Sons, Inc., New York, 1961, xviii + 381 p., 24 cm. Price $9.50.

This attractive newcomer to the ranks of the textbooks on methods of mathematical physics comes to us directly from Moscow (where, for the past four years, the author has been an Associate Professor of Mathematics at the University of Idaho). This book contains material which has been used in the author's classes to seniors and beginning graduate students in mathematics, applied mathematics, physics, and engineering for the past five years. The author's stated purpose is not to present a vast number of seemingly unrelated mathematical techniques and tricks that are used in the mathematical treatment of problems which arise in